

# Exam

## *Advanced Public Finance*

Wirtschaftswissenschaftliche Fakultät der Friedrich-Schiller-Universität Jena  
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First name:		Last name:	
Student ID number:		Course of study:	

**Please note:**

- (a) The exam consists of 9 pages including this one. Please check whether your copy of the exam is complete.
- (b) The exam consists of 3 questions. The maximum number of points is 60. You have 60 minutes to complete the exam.
- (c) Please answer the questions by writing into the boxes provided after each question. **Do not use your own paper!** Fill your name and student ID number into the form at the top of each page.
- (d) If not defined otherwise, variables have the same meaning as in class. Please make sure that your answers are clearly legible and without any ambiguity. Your answers have to be tractable. If you use diagrams, make sure to label and explain them.
- (e) You may use a calculator, but it must not have a text storage function. You may use a dictionary, but it must not contain any notes.
- (f) It is your own responsibility to hand in your copy of the exam to the supervisory staff at the end of the exam.

Question	1	2	3	Sum	Grade
Points receivable	20	20	20	60	
Points received					

**Question 1: Private Provision of Public Goods** (20 Points)

The following model has been discussed in class:

There are  $H$  households ( $H \geq 2$ ), with  $h = 1, \dots, H$ , who contribute to the provision of a public good  $G$ .

Notation

- $G$  : Privately provided quantity of the public good
- $g^h$  : Contribution to  $G$  of household  $h$
- $G^{-h}$  : Contributions to  $G$  of all households but  $h$
- $x^h$  : Consumption of the private good by household  $h$
- $w^h$  : Income of household  $h$

The household maximizes the utility function

$$\max_{x^h, g^h} U^h = x^h G \quad (1)$$

subject to

$$w^h = x^h + g^h \quad (2)$$

$$G = g^h + G^{-h} \quad (3)$$

$$g^h \geq 0 \quad (4)$$

with the assumption that all prices are normalized to 1.

- (a) State the maximization problem of household  $h$  when there is private provision using (1), (2) and (3). (Do not solve it!) (2 Points)

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

The reaction function following from the household's maximization problem is

$$g^h = \max \left( \frac{w^h - G^{-h}}{2}; 0 \right) \quad (5)$$

- (b) Derive the reaction function (5) from the household's maximization problem – see question (a). (6 Points)

- (c) If  $g$  represented consumption of another private good (beside  $x$ ), explain how the household would divide income between  $x$  and  $g$ . Compare this with the situation reflected in the reaction function (5). No calculations are required! (4 Points)

Name:

Student ID:

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In symmetric equilibrium, all households together contribute to the public good

$$G^{priv} = \frac{H}{H+1}w^h \quad (6)$$

The socially optimal quantity of the public good is

$$G^{opt} = \frac{Hw^h}{2} \quad (7)$$

- (d) Compare the total private contribution  $G^{priv}$  with the socially optimal contribution  $G^{opt}$ .
- Why can one say that there is a market failure with private provision?
  - How does the market failure depend on  $H$ ?
  - What is the underlying reason (explain)?

(8 Points)

**Question 2: Short Questions** (20 Points)**Question 2(a) Inverse Elasticity Rule** (10 Points)

- (a1) The first-best optimal indirect taxation would be to tax all goods in a uniform way. Briefly explain why this would be first-best and why this is normally not possible. (4 Points)

The following shows the so called Generalized Inverse Elasticity Rule for the case with  $h, h = 1 \dots H$ , (potentially) different households (notation as in class):

$$\tau_k = \left( 1 - \frac{\sum_{h=1}^H W^h \lambda^h x_k^h}{\mu \sum_{h=1}^H x_k^h} \right) \frac{1}{\eta_{kk}} \quad (8)$$

This rule determines the optimal (second-best) tax rate for good  $k$ ,  $\tau_k$ .

- (a2) Describe the two main aspects which are captured by this rule. Where can you see them in (8)? Illustrate the possible trade-off because of these two aspects with an example. (6 Points)

**Question 2(b) Optimal Non-Linear Taxation** (10 Points)

To derive the optimal non-linear tax schedule with households of different ability, we studied the following governmental maximization problem:

$$\max_{(x^H, x^L, y^H, y^L)} U^H(x^H; y^H) \quad (9)$$

subject to

$$U^L(x^L; y^L) \geq \bar{U}^L \quad (10)$$

$$U^H(x^H; y^H) \geq U^H(x^L; y^L) \quad (11)$$

$$U^L(x^L; y^L) \geq U^L(x^H; y^H) \quad (12)$$

$$(y^H - x^H)N^H + (y^L - x^L)N^L \geq 0 \quad (13)$$

with  $N^i$ : number of households of type  $i = L, H$ ;  $x^i$ : consumption;  $y^i$ : income.

- (b1) Briefly explain the objective function and each of the four constraints.  
(4 Points)

- (b2) One of the results of this optimization problem is described as “No Distortion at the Top”. Briefly explain what this means! (2 Points)

Name: \_\_\_\_\_

Student ID: \_\_\_\_\_

(b3) What is the analogous result for the other household type? Why are the two household types not treated the same? (4 Points)

**Question 3: Multiple Choice Questions** (20 Points)

You will be *awarded one point* for ticking a correct statement and for not ticking an incorrect statement. You will *neither receive nor lose points* for marking statements incorrectly.

**Question 3(a) Cross-country observations** (3 Points)

- Denmark has a large welfare state compared to most other countries. Because of this, social security contributions play an important role for public revenues.
- A larger part of public expenditure is spent on defense in the United States than in Italy.
- Property taxes are an important source of tax revenues in Germany.

**Question 3(b) Tax evasion** (6 Points)

- Tax evasion is the intentional failure to declare taxable economic activity, while the illegal reorganization of economic activity is called tax avoidance.
- Declared income increases both with a higher gross fine rate and a higher probability of detection.
- With decreasing absolute risk aversion, higher income leads to a higher absolute level of tax evasion. The reason is that higher income makes individuals less risk averse.
- The optimal real-world policy against tax evasion implies a positive level of tax evasion.

In the paper by Marion and Muehlegger (2008), there are two possible uses of diesel fuel: business/transport, which is taxed, and residential (for heating homes), which is untaxed. In 1993, the government added red dye to residential diesel.

- Absent evasion, a change in either the tax-exclusive price or the tax rate should reduce sales of taxed diesel equally.
- If evasion exists, we would expect sales of taxed diesel to respond less to a change in taxes than to a change in prices.

**Question 3(c) Impure and pure public goods** (5 Points)

- The Samuelson Rule says: The provision of the public good is optimal when the marginal rate of substitution (MRS) is equal to the sum of the marginal rate of transformation (MRT).
- With an impure public good, there is rivalry with many users, which reduces the return the public good gives to each user.



- To internalize the congestion externality of an impure public good, a user charge equal to this externality (at the point of optimal usage) has to be implemented. It follows that for a pure public good, such a user charge would not be sufficient to finance its provision.
- For a usage cost function, which is homogeneous of degree  $\lambda > 0$ , the revenues from a user charge meant to internalize the congestion externality exceed the costs of providing the impure public good.
- When a public good is financed by a distortionary commodity taxation, there is overprovision compared to a non-distorting tax-financing.

**Question 3(d) Public provision of private goods (3 Points)**

- Cash transfers are, in general, superior to in-kind transfers for the consumers because they do not constrain the consumers at some undesired level of consumption.
- The publicly provided private good slackens the self-selection constraint of optimal direct taxation with unobservable individual productivity if the high-productive and the low-productive individual want to consume the same level of the good provided in-kind.
- About 70% of social expenditure in Germany and Italy is provided in-kind.

**Question 3(e) Excess burden (3 Points)**

- Excess burden of taxation is the utility loss which the tax-payer suffers from only if he continues to consume the taxed good.
- With non-uniform taxes, the relative prices of goods change. The substitution effect leads to an excess burden.
- The excess burden increases proportionally with the tax rate.